SOLID PROPELLANT COMBUSTION IN THE PRESENCE OF PHOTOIRRADIATION

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Previous studies of the nonsteady processes associated with the irradiation of propellants with light have chiefly been devoted to the question of ignition [1-3].* It is also important to consider the effect of such an easily controlled influence as light on the propellant combustion process. We have attempted to estimate the dependence of the propellant burning rate on the intensity of the luminous radiation. Cases of steady-state combustion and combustion in the presence of a light flux varying harmonically with time are considered. It is assumed that the incident light flux is absorbed in the solid phase in accordance with the Bouguer–Lambert exponential law with constant transparency index. Steady-state combustion is considered within the framework of the Zel'dovich theory [4]. It is shown that in the steady state irradiation is equivalent to a certain increase in the initial temperature of the propellant. In the case of combustion with irradiation this makes it possible to use the data on steady-state combustion without irradiation. Nonsteady combustion in the presence of a periodically varying light flux is described with the aid of the Novozhilov model [5]. A correction to the mean burning rate ($\Delta u°$), proportional to the square of the light flux amplitude, is obtained. In the case of an exponential dependence of the burning rate on initial temperature the correction $\Delta u°$ is negative. The effect of irradiation on the stability of the steady-state propellant combustion mode is discussed.

1. Steady-State Irradiation. Combustion Law

The equation describing the change of temperature in the solid phase in the presence of conductive and radiative heat transfer takes the form

$$\rho c\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}+J(t)e^{\sigma x}\right)\quad(-\infty<x\leq 0)\tag{1.1}$$

where J(t) is the fraction of the incident luminous flux (cal/sec · cm^2) absorbed in the solid phase; σ is the transparency index of the solid phase in the Bouguer–Lambert law (1/cm); ρ, c, and λ are the density, specific heat, and thermal conductivity of the solid phase, respectively; u is the propellant burning rate.

In this case the following boundary conditions must be satisfied:

$$T(-\infty)=T_0,\ T(0)=T_1\tag{1.2}$$

where T_0 is the initial temperature of the propellant, and T_1 is the temperature of the propellant surface.

In the steady-state case Eq. (1.1) is easily integrated and its solution takes the form [6]

$$T-T_0=(T_1°-T_0)\exp\frac{u°}{\varkappa}x+\frac{J°}{\lambda(u°/\varkappa-\sigma)}\left(e^{\sigma x}-\exp\frac{u°}{\varkappa}x\right),\quad \varkappa=\frac{\lambda}{\rho c}\tag{1.3}$$

*P. F. Pokhil, "Mechanism of combustion of colloidal powders," Doctoral Dissertation, Pt. 3, Institute of Chemical Physics, Academy of Sciences of the USSR, Moscow (1954).

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If the transparency of the propellant is low ($\sigma \varkappa / u^\circ \gg 1$), then in absolute value solution (1.3) tends to the Michelson distribution

$$T - T_0 = (T_1^\circ - T_0) \exp\left(\frac{u^\circ}{\varkappa} x\right) \tag{1.4}$$

At any value of σ the temperature curve (1.3) passes above the Michelson distribution (1.4).

The steady-state surface temperature gradient f° is related with the initial temperature T_0 by the expression

$$\varkappa f^\circ = u^\circ (T_1^\circ - T_0) - J^\circ / \rho c \tag{1.5}$$

which in the absence of irradiation ($J^\circ = 0$) goes over into the usual relation corresponding to distribution (1.4)

$$\varkappa f_0^\circ = u_0^\circ (T_{10}^\circ - T_0) \tag{1.6}$$

We note that for steady-state irradiation the gradient f° does not depend on σ.

Since for steady-state combustion the gradient f° cannot be negative, from (1.5) there follows the upper bound on the value of the light flux J°, at which the steady-state mode is possible,

$$\frac{J^\circ}{u^\circ \rho c (T_1^\circ - T_0)} < 1 \tag{1.7}$$

In particular, for the Zel'dovich model, in which T_1 is constant, and a dependence of u_0° on T_0 of the type $u_0^\circ \sim e^{\beta T_0}$ there is a maximum burning rate u_M° that is obtained at an initial temperature T_0 close to the surface temperature T_1. In this case ratio (1.7) tends to unity, and $u^\circ \to u_M^\circ$ for finite light fluxes J°. Moreover, the ratio (1.7) should not be too close to unity; otherwise the smallness of the gradient f° leads to a powerful expansion of the solid-phase reaction zone, so that the Zel'dovich combustion model is inapplicable. In this case the heated surface layer of propellant may periodically explode in accordance with a thermal mechanism.

Now, in the absence of irradiation let the dependence of the burning rate u_0° and surface temperature T_{10}° on the initial temperature T_0 and pressure p be known:

$$u_0^\circ = F(T_0, p), \quad T_{10}^\circ = G(T_0, p) \tag{1.8}$$

Using relation (1.6), we can express u_0° and T_{10}° in terms of p and f_0°. In accordance with Novozhilov's theory [5] the steady-state laws $u(p, f)$, $T_1(p, f)$ obtained are also valid in the nonsteady case.

We now assume that the relations $u(p, f)$ and $T_1(p, f)$ obtained for $J = 0$ are also valid in the presence of a light flux. Then, substituting relation (1.6) in the dependence $u_0^\circ = F(p, T_0)$ and replacing f_0° by the expression for f° from (1.5), we obtain the dependence of the steady-state burning rate u° on the initial temperature T_0 in the presence of irradiation

$$u^\circ = F\left(p, T_0 + \frac{J^\circ}{u^\circ \rho c}\right) \tag{1.9}$$

Thus, the increase in burning rate from u_0° to u° in the presence of irradiation of the propellant surface by a steady light flux is equivalent to the change in burning rate associated with an increase in initial temperature by the amount

$$\Delta T_0 = J^\circ / u^\circ \rho c \tag{1.10}$$

Starting from this, the burning rate u° in the presence of irradiation may conveniently be found as follows (Fig. 1): we construct graphs of the functions $F^{-1}(u)$ and $(T_0 + J^\circ / u \rho c)$ in the coordinate system (u, T); the point of intersection of the graphs determines the unknown burning rate u° and the effective initial temperature T_0^*

$$T_0^* = T_0 + J^\circ / u^\circ \rho c \tag{1.11}$$

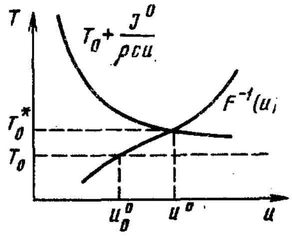

Fig. 1

The equivalence of a light flux and initial temperature in the case of sufficiently transparent propellants was previously noted in [6].

Equation (1.10) makes it possible to determine the rate fluxes necessary to attain a given level of the burning rate u° for a known dependence of $u_0°$ on initial temperature

$$J° = \Delta T_0 \rho c u°$$

Thus, in accordance with the experimental data of [7] for a ballistite (N powder) at a pressure p = 1 atm, in order to raise the burning rate from 0.6 (at T_0 = 0°C) to 1.02 mm/sec a light flux of 2.8 cal/cm^2 · sec is required, which corresponds to raising the initial temperature T_0 to T_0^* = 50°C. At a pressure p = 20 atm in order to raise the burning rate from 0.26 cm/sec (T_0 = 0°C) to 0.35 cm/sec (T_0^* = 50°C) a light flux J° = 7.3 cal/cm^2 · sec is required.

At low values of the illuminance $[J°/u° \rho c (T_1° - T_0) \ll 1]$ we have the following approximate expression for u°, calculated correct to a factor proportional to the square of the light flux J°,

$$u° = u_0° \left[1 + k\delta + {}^1/_2 (l - 2k^2)\delta^2\right] \tag{1.12}$$

where $u_0°$ is the steady-state burning rate in the absence of irradiation

$$\delta = \frac{J°}{u_0° \rho c (T_{10}° - T_0)}, \quad k = (T_{10}° - T_0)\frac{\partial \ln u_0°}{\partial T_0}, \quad l = \frac{(T_{10}° - T_0)^2}{u_0°}\frac{\partial^2 u_0°}{\partial T_0^2} \tag{1.13}$$

Here, as before, a degree sign denotes the steady-state value, while a zero subscript denotes that the corresponding quantity is taken at J° = 0.

2. Stability of Steady-State Combustion in the Presence of Irradiation

The equivalence of steady-state irradiation of the propellant surface to a certain increase in initial temperature makes it possible to employ, in combustion problems with irradiation, the data on steady-state combustion without irradiation. This is achieved by simple conversion of the true initial temperature T_0 to the effective temperature T_0^*.

However, in nonsteady combustion modes in the presence of irradiation relation (1.5) ceases to hold and the temperature gradient at the propellant surface depends on the transparency index of the solid phase σ. In this connection, in the general case such characteristics of the propellant as the stability limit of the steady-state combustion mode and the natural frequency depend on σ and J°.

Nonetheless, if the transparency of the propellant is low and the light is absorbed in a narrow surface layer of the solid phase, whose width is negligible as compared with the width of the solid-phase induction zone ($\sigma \varkappa / u° \gg 1$), the heat distribution in the solid phase in the presence of irradiation has the same form as in the absence of irradiation. In this limiting case the equations for the stability limit of steady-state combustion modes and the natural frequency of the propellant, obtained in the absence of irradiation [4,5], retain their previous form. Thus, for example, for the case of constant surface temperature stable steady-state modes correspond to the condition [4]

$$k^* < 1 \tag{2.1}$$

where k* is the Zel'dovich criterion calculated for $T_0 = T_0^*$

$$k^* = k(T_0^*) = (T_1 - T_0^*)\frac{\partial \ln u°}{\partial T_0^*} \tag{2.2}$$

For a $u_0°(T_0)$ dependence of the type $u_0° \sim \exp \beta T_0 (k = \beta (T_1 - T_0))$ the presence of irradiation leads to a decrease in the value of the criterion k, i.e., greater combustion stability.

In the presence of a variable surface temperature stable steady-state modes satisfy the condition [5]

$$r^* > (k^* - 1)^2/(k^* + 1) \tag{2.3}$$

where r^* is the Novozhilov parameter for $T_0 = T_0^*$

$$r^* = \partial T_1^\circ / \partial T_0^* \tag{2.4}$$

In this case the presence of irradiation, like an increase in initial temperature, may lead either to an increase or to a decrease in combustion stability. For example, for N powder, in accordance with the data of [7], irradiation brings the steady-state combustion mode closer to the stability limit (2.3). We note that conditions (2.1) and (2.3) are also confirmed by an exact analysis of the problem of the natural frequency of a propellant on the assumption that its transparency is low.

3. Periodic Irradiation. Quasi-Linearization of the Problem

We will now consider a propellant burning in the presence of periodically varying irradiation of the surface on the assumption that the pressure is constant. Let the mean value of the light flux be equal to J°, so that

$$J = J^\circ + \Delta J \cos \omega t \tag{3.1}$$

and let the burning rate and surface temperature in the steady-state mode at $J = J^\circ$ be equal to u° and T_1°, respectively. We introduce the dimensionless variables

$$\theta = \frac{T - T_0^*}{T_1^\circ - T_0^*}, \quad \xi = \frac{u^\circ}{\varkappa} x, \quad \tau = \frac{(u^\circ)^2}{\varkappa} t$$
$$\varphi = \left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=0}, \quad v = \frac{u}{u^\circ}, \quad \eta = \frac{J}{u^\circ \rho c (T_1^\circ - T_0^*)} \tag{3.2}$$

In these variables the problem is formulated as follows: for a given law of variation of the light flux

$$\eta(\tau) = \alpha + \varepsilon \cos \gamma\tau \tag{3.3}$$

and known functions

$$v = v(\varphi), \quad \vartheta = \vartheta(\varphi) \quad \left(\vartheta = \frac{T_1 - T_1^*}{T_1^\circ - T_0^*}\right) \tag{3.4}$$

to find $v = v(\tau)$, if the gradient φ (3.2) satisfies the equation

$$\frac{\partial \theta}{\partial \tau} + v \frac{\partial \theta}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{\partial \theta}{\partial \xi} + \eta(\tau) e^{\nu\xi}\right) \tag{3.5}$$

with boundary conditions

$$\theta|_{\xi \to -\infty} = -\alpha, \quad \theta|_{\xi=0} = \vartheta \tag{3.6}$$

Here we have introduced the notation

$$\alpha = \frac{J^\circ}{u^\circ \rho c (T_1^\circ - T_0^*)}, \quad \varepsilon = \frac{\Delta J}{u^\circ \rho c (T_1^\circ - T_0^*)}, \quad \nu = \frac{\sigma \varkappa}{u^\circ}, \quad \gamma = \frac{\omega \varkappa}{(u^\circ)^2} \tag{3.7}$$

System (3.3)-(3.6) makes it possible to find the dependence of the burning rate and surface temperature on time. With the object of finding only the first correction of the order of ε^2 to the mean burning rate, we seek the solution of the problem in the form of successive approximations

$$\begin{gathered}
v(\tau) = 1 + \varepsilon v_1(\tau) + \varepsilon^2 v_2(\tau) + \ldots \\
\vartheta(\tau) = 1 + \varepsilon \vartheta_1(\tau) + \varepsilon^2 \vartheta_2(\tau) + \ldots \\
\theta(\tau, \xi) = \theta_0(\xi) + \varepsilon \theta_1(\tau, \xi) + \varepsilon^2 \theta_2(\tau, \xi) + \ldots \\
\varphi(\tau) = \frac{\partial \theta}{\partial \xi}\Big|_{\xi=0} = 1 + \varepsilon \frac{\partial \theta_1}{\partial \xi}\Big|_{\xi=0} + \varepsilon^2 \frac{\partial \theta_2}{\partial \xi}\Big|_{\xi=0} + \ldots
\end{gathered} \tag{3.8}$$

Substituting expansions (3.8) in Eq. (3.5), boundary conditions (3.6), and relations (3.4), we obtain a sequence of systems of equations for finding the successive approximations. For the first approximation the system of equation takes the form

$$\frac{\partial\theta_1}{\partial\tau} + \frac{\partial\theta_1}{\partial\xi} - \frac{\partial^2\theta_1}{\partial\xi^2} + v_1\frac{\partial\theta_0}{\partial\xi} = \nu\cos(\gamma\tau)e^{\nu\xi}$$
$$v_1 = a_1\varphi_1, \quad \varphi_1 = \frac{\partial\theta_1(\tau,0)}{\partial\xi} \tag{3.9}$$
$$\theta_1(\tau,0) = \vartheta_1 = b_1\varphi_1, \quad \theta_1(\tau,-\infty) = 0$$

The second approximation satisfies the system

$$\frac{\partial\theta_2}{\partial\tau} + \frac{\partial\theta_2}{\partial\xi} - \frac{\partial^2\theta_2}{\partial\xi^2} + v_2\frac{\partial\theta_0}{\partial\xi} = -v_1\frac{\partial\theta_1}{\partial\xi}$$
$$v_2 = a_1\varphi_2 + {}^1/_2 a_2\varphi_1^2, \quad \varphi_2 = \partial\theta_2(\tau,0)/\partial\xi \tag{3.10}$$
$$\theta_2(\tau,0) = \vartheta_2 = b_1\varphi_2 + {}^1/_2 b_2\varphi_1^2, \quad \theta_2(\tau,-\infty) = 0$$

In systems (3.9) and (3.10) the coefficients a_1, a_2, b_1, and b_2 are given by relations (3.4)

$$a_1 = \left.\frac{\partial v}{\partial\varphi}\right|_{\varphi=1}, \quad a_2 = \left.\frac{\partial^2 v}{\partial\varphi^2}\right|_{\varphi=1}$$
$$b_1 = \left.\frac{\partial\vartheta}{\partial\varphi}\right|_{\varphi=1}, \quad b_2 = \left.\frac{\partial^2\vartheta}{\partial\varphi^2}\right|_{\varphi=1} \tag{3.11}$$

4. Correction to Mean Burning Rate

Omitting for the time being finding an explicit expression for θ_1, we seek the constant component $v_2°$, which determines the correction to the mean burning rate. From system (3.10) it follows that $v_2°$ depends on the steady-state component of the second approximation for the temperature $\theta_2°(\xi)$. Integrating the first equation in system (3.10) with respect to ξ from $-\infty$ to 0, we obtain the relation

$$\vartheta_2° = \theta_2°(0) = \frac{\partial\theta_2°(0)}{\partial\xi} - v_2°\theta_0(0) - [v_1\theta_1]°|_{\xi=0} \tag{4.1}$$

Together with the expressions for $v_2°$ and $\vartheta_2°$ from (3.10) relation (4.1) forms a closed system of algebraic equations. Solving this system, we obtain the expression for $v_2°$

$$v_2° = A\left[\frac{a_1^2 b_1 + {}^1/_2(a_2 - b_1 a_2 + a_1 b_2)}{1 - a_1 - b_1}\right] \tag{4.2}$$

where A is determined by the steady-state component of the square of the first approximation for the temperature gradient φ_1

$$A = [(\varphi_1)^2]° = \left[\left(\frac{\partial\theta_1(\tau,0)}{\partial\xi}\right)^2\right]° \tag{4.3}$$

In its turn, the gradient φ_1 is determined by the solution of system (3.9), which depends on the zeroth approximation of the temperature derivative $d\theta_0/d\xi$. From the steady-state distribution (1.3) we find

$$\frac{d\theta_0}{d\xi} = \left(1 - \frac{\alpha\nu}{1-\nu}\right)e^{\xi} + \frac{\alpha\nu}{1-\nu}e^{\nu\xi} \tag{4.4}$$

Substituting (4.4) in system (3.9) and assuming that the natural oscillations of the system are damped, we find that θ_1 can be represented in the form

$$\theta_1 = \mathrm{Re}\,[e^{i\gamma\tau}(A_1 e^{\mu\xi} + A_2 e^{\xi} + A_3 e^{\nu\xi})] \tag{4.5}$$

where

$$\mu = \mu_1 + i\mu_2 = ({}^1/_2 + \gamma/R) + i\,{}^1/_2 R$$
$$R = [{}^1/_2(\sqrt{1+16\gamma^2} - 1)]^{1/2} \tag{4.6}$$

The coefficients A_1, A_2, and A_3 satisfy the system of equations

$$\mu g A_1 + (g + i\gamma)A_2 + \nu A_3 = 0 \tag{4.7}$$
$$\mu h A_1 + h A_2 + (\nu - \nu^2 - \nu h + i\gamma)A_3 = \nu$$
$$(1 - \mu b_1)A_1 + (1 - b_1)A_2 + (1 - \nu b_1)A_3 = 0$$

where

$$g = a_1\left(1 - \frac{\alpha\nu}{1-\nu}\right), \quad h = a_1\frac{\alpha\nu}{1-\nu} \tag{4.8}$$

From (4.5) and (4.7) we find the temperature gradient at the surface φ_1

$$\varphi_1 = \nu \sqrt{\frac{(\nu-\mu_1)^2+\mu_2^2}{\Delta_1^2+\Delta_2^2}} \cos(\gamma\tau+\psi) \tag{4.9}$$

$$\operatorname{tg}\psi = \frac{(\nu-\mu_1)\Delta_1-\mu_2\Delta_2}{(\nu-\mu_1)\Delta_2+\mu_2\Delta_1} \tag{4.10}$$

where, correct to the factor γ, Δ_1, and Δ_2 are respectively equal to the real and imaginary parts of the determinant of system (4.7)

$$\begin{aligned} \Delta_1 &= \gamma(\mu_1 b_1 - 1) + \mu_2 a_1 + \frac{\nu-\nu^2}{\gamma}[\gamma\mu_2 b_1 + g(1-\mu_1)] \\ \Delta_2 &= \gamma\mu_2 b_1 + (1-\mu_1-\alpha\nu)a_1 + \frac{\nu-\nu^2}{\gamma}[\gamma(1-\mu_1 b_1) - \mu_2 g] \end{aligned} \tag{4.11}$$

From (4.9), (4.2), and (4.8) we obtain the expression for the mean burning rate increment Δu°

$$\frac{\Delta u^\circ}{u^\circ} = \frac{1}{2}\left(\frac{\Delta J}{u^\circ\rho c(T_1^\circ - T_0^*)}\right)^2 \nu^2 \frac{(\nu-\mu_1)^2+\mu_2^2}{\Delta_1^2+\Delta_2^2} \frac{a_1^2 b_1 + {}^1/_2(a_2 - b_1 a_2 + a_1 b_2)}{1-a_1-b_1} \tag{4.12}$$

The expression for the increment Δu° contains derivatives of the burning rate u and surface temperature T_1 with respect to the temperature gradient at propellant surface φ. However, in practice, it is usual to employ the steady-state dependences of the burning rate u° and temperature T_1° on the initial temperature T_0 obtained in the absence of irradiation (1.8). Accordingly, we shall express the derivatives $a_1, \ldots, b_2$ in terms of derivatives of the functions F and G. As noted above, the functions $u(f)$ and $T_1(f)$ are given implicitly by the system

$$u = F\left(T_1 - \frac{\varkappa f}{u}\right), \quad T_1 = G\left(T_1 - \frac{\varkappa f}{u}\right) \tag{4.13}$$

Using system (4.13) and the expression for the gradient f° in the presence of steady-state irradiation (1.5), we obtain

$$\begin{aligned} a_1 &= \left(\frac{d\ln u}{d\ln f}\right)_{f=f^\circ} = \frac{k^*}{k^*+r^*-1} \\ b_1 &= \frac{f}{(T_1^\circ - T_0^*)}\left(\frac{dT_1}{df}\right)_{f=f^\circ} = \frac{k^*}{k^*+r^*-1} \end{aligned} \tag{4.14}$$

$$\begin{aligned} a_2 &= \frac{(f^\circ)^2}{u^\circ}\left(\frac{d^2u}{df^2}\right)_{f=f^\circ} = \frac{1}{(k^*+r^*-1)^2}\left[k^{*\prime} - \frac{k^*(k^{*\prime}+r^{*\prime})}{k^*+r^*-1} + k^*(1-r^*)\right] \\ b_2 &= \frac{(f^\circ)^2}{(T_1^\circ - T_0^*)}\left(\frac{d^2T_1}{df^2}\right)_{f=f^\circ} = \frac{1}{(k^*+r^*-1)^2}\left[r^{*\prime} - \frac{r^*(k^{*\prime}+r^{*\prime})}{k^*+r^*-1} - k^* r^*)\right] \end{aligned} \tag{4.15}$$

where

$$k^* = \frac{(T_1^\circ - T_0^*)}{u^\circ}\frac{dF}{dT_0^*}, \quad r^* = \frac{dG}{dT_0^*} \tag{4.16}$$

$$k^{*\prime} = (T_1^\circ - T_0^*)\frac{dk^*}{dT_0^*}, \quad r^{*\prime} = (T_1^\circ - T_0^*)\frac{dr^*}{dT_0^*} \tag{4.17}$$

The relation between the derivatives with respect to the gradient and the derivatives with respect to initial temperature in the presence of irradiation has the same form as in the absence of irradiation [5]. The only difference consists in the fact that the parameters k and r must be calculated not at the true initial temperature T_0 but at the effective temperature T_0^*. Substituting (4.16), (4.17) in the expression for Δu°, we obtain

$$\frac{\Delta u^\circ}{u^\circ} = \frac{1}{4}\left(\frac{\Delta J}{u^\circ\rho c(T_1^\circ - T_0^*)}\right)^2 \nu^2 \frac{(\nu-\mu_1)^2+\mu_2^2}{\Delta_1^2+\Delta_2^2} \frac{k^{*\prime} - k^*(k^*+r^*-1)}{(k^*+r^*-1)^2} \tag{4.18}$$

The sign of the increment coincides with the sign of the last fraction in expression (4.18). If the dependence of the burning rate on initial temperature is represented in the form $u^\circ \sim e^{\beta T_0}$, where β is constant, then

$$k^* = (T_1^\circ - T_0^*)\beta, \quad k^{*\prime} = k^*(r^* - 1)$$

and the correction Δu° is negative. This result is similar to that obtained from an analysis of combustion in the presence of a variable heat flux [8]. The physical explanation is based on the fact that the effect of additional heating of the propellant is weakened as the mass flux increases, whereas the cooling effect is intensified as the mass flux decreases. The dependence of Δu° on the frequency γ and the transparency index ν is expressed by the factor

$$\nu^2 \frac{(\nu - \mu_1)^2 + \mu_2^2}{\Delta_1^2 + \Delta_2^2} \tag{4.19}$$

In accordance with (4.6) and (4.11) at high frequencies $\gamma \gg 1$ and a transparency index ν close to zero or comparable with unity [so that $\gamma^{-1}(\nu - \nu^2) \ll 1$], for (4.19) we have the asymptotic expression [correct to $o(1/\gamma)$]

$$\nu^2 \frac{\gamma - (\nu - 1/2)\sqrt{2\gamma}}{[\gamma\sqrt{1/2\gamma} + \gamma(1/2 b_1 - 1) + a_1\sqrt{1/2\gamma}]^2 + [\gamma\sqrt{1/2\gamma}\, b_1 + (1/2 - \alpha\nu - \sqrt{1/2\gamma})\, a_1]^2} \tag{4.20}$$

It follows from (4.20) that in the limit as $\gamma \to \infty$ the correction Δu° tends to zero. In absolute magnitude Δu° is the greater, the greater the transparency index ν.

In the other limiting case as $\gamma \to 0$ and simultaneously $\gamma \ll (\nu - \nu^2)$ expression (4.19) tends to

$$(\nu - 1)^2 [\alpha a_1 + (\nu - 1)(1 - b_1 - g)]^{-2} + O(\gamma^2) \tag{4.21}$$

The value of the correction $\Delta u^\circ / u^\circ$ can reach values comparable with unity only at sufficiently large amplitudes of the light flux oscillations (ε close to unity). At moderate amplitudes ΔJ, the correction $\Delta u^\circ / u^\circ$ is not very great. Thus, for example, for N powder, whose transparency index σ is equal to 15 cm^{-1} over a broad range of light waves,* we find that at $p = 1$ atm, $T_0 = -100$°C, $J^\circ = 8.5$ cal/cm^2 · sec, $\Delta J = 8$ cal/cm^2, $\omega = 5$–10 Hz the correction $|\Delta u^\circ / u^\circ|$ is about 10%.

Near the stability limit of the steady-state combustion mode and at frequencies γ close to the natural frequency of the propellant the correction rapidly increases. In this case fairly large changes in the mean burning rate, possibly leading to extinction, are to be expected in the presence of periodic irradiation.

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